

# Synthesis of Octave-Band Quarter-Wave Coupled Semi-Tracking Stripline Junction Circulators

J. Helszajn, *Fellow, IEEE*

**Abstract**—The complex gyrator circuit of stripline circulators using tracking junctions which satisfy the physical and magnetic variables of the Wu and Rosenbaum boundary conditions, is a nearly frequency independent octave-band resistive network. Such a junction may exhibit two minimas in its reflection coefficient when matched by a two section impedance transformer. However, a third minima may be realised by utilizing a complex rather than a real gyrator circuit. This paper summarises this class of semi-tracking solution in a simple way as a preamble to the design of a degree-3 quarter wave coupled circulator. The overall frequency behaviour of this class of junction has been separately evaluated by combining the electromagnetic and network problems (in conjunction with a two step impedance matching network).

## I. INTRODUCTION

THE classic junction circulator using a planar disk resonator exhibits a host of solutions; some of which are well behaved and others which are not. A particularly attractive one is the so called tracking one for which the gyrator circuit is a nearly frequency independent conductance over approximately an octave frequency band [1]–[5]. However, it is not an optimum solution in so far as the network problem is concerned since a complex rather than a real gyrator circuit is to be preferred. Such a field of semi-tracking solutions can in fact be realized in the vicinity of the tracking one by perturbing either the coupling angle of the junction or its magnetization.

The two most important parameters in the synthesis of semi-tracking circulators are the loaded  $Q$ -factor of the junction and the frequency interval over which the complex gyrator circuit exhibits a nearly frequency independent conductance and a nearly constant susceptance slope [6]–[9]. Once these quantities are fixed (by the coupling angle of the resonator and its magnetization), it is merely necessary to adjust the absolute levels of the real and imaginary parts of the junction (with the aid of the ground plane spacing) to meet the required specification. A description of the junction in terms of its complex gyrator admittance and loaded  $Q$ -factor is therefore necessary and sufficient. Although no closed form solution is in general available it may be derived graphically from a knowledge of the frequency response of the device. A detailed investigation of this problem indicates that a host of semi-tracking solutions may be realized in the vicinity of the tracking one by properly

adjusting the details of the junction. One possibility is the design of a degree-3 equal ripple frequency response over one octave [9].

Since the design procedure outlined in this paper assumes an idealized gyrator circuit, the actual load conditions have been evaluated by combining the electromagnetic and network problems using the eigenvalue approach [8]. An octave band design using two quarter-wave transformers based on this model predicts a worst VSWR of 1.11 over the band instead of a network value of 1.05. A circulator with a VSWR of 1.20 at the bandedges has been constructed.

A feature of quarter-wave coupled circulators using semi-tracking junctions is that the relative dielectric constant of the transformer regions is small compared to that of the ferrite material. This is a somewhat fortunate situation in that it ensures that the usual assumed magnetic wall boundary condition between the circulator ports is consistent with practice and ensures fair agreement between theory and experiment.

## II. NETWORK PROBLEM

It is now well understood that stripline junction circulators may be adjusted to exhibit 1-port complex gyrator circuits for which the real part (conductance) is nearly frequency independent and the imaginary part (susceptance) has a small but non-zero value. Such circuits may exhibit three minimas in their frequency responses when matched by a two section impedance transformer. The topology of this network and its frequency response are illustrated in Fig. 1(a) and (b). The relationship between the frequency response of the network and the circuit elements has been discussed in [6]. Table I summarizes the relationship between  $\text{VSWR}(\text{max})$ , the elements of the gyrator circuit (susceptance slope parameter  $b'$ , conductance  $g$ , loaded  $Q$ -factor  $Q_L$ ), the matching circuit (admittances  $y_1$  and  $y_2$ ) for an octave band specification ( $2\delta_0 = 0.66$ ) with  $\text{VSWR}(\text{min})$  equal to 1. The electromagnetic problem (in the next section) is to obtain a one to one correspondence between the entries of this table and the planar junction configuration.

A recent, more general matching theory, suggests that a precise realization of the susceptance slope parameter may not be as critical as historically supposed provided the minimas in the reflection coefficient of the device are not forced to pass thru zero [9]. Table II displays the range of specifications realizable with a degree –3 network with  $2\delta_0 = 0.66$ . It is obtained by taking the value of conductance given by the tracking solution [2]–[5] as an initial trial value and allowing

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The author is with Heriot-Watt University, Edinburgh, Scotland.  
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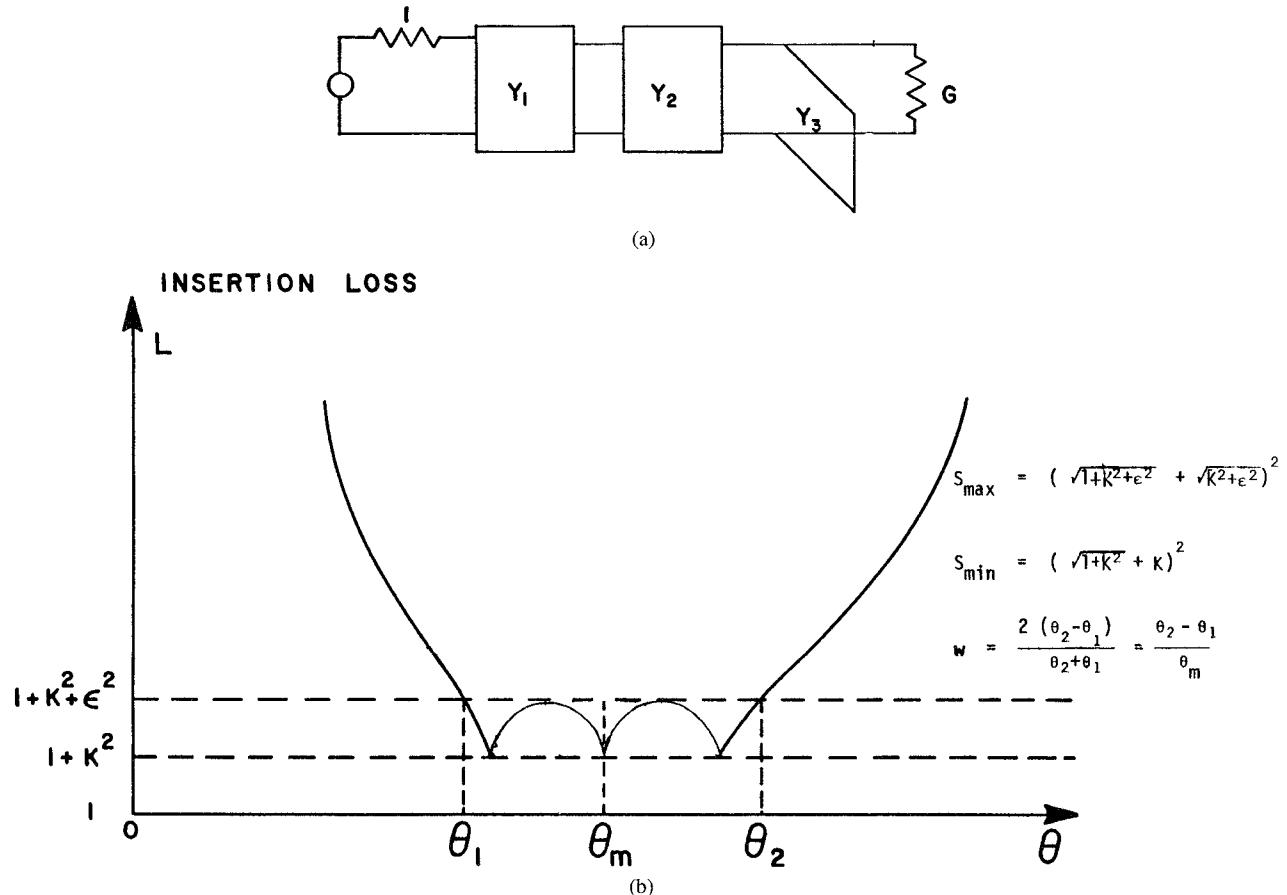


Fig. 1. (a) Topology of quarter-wave coupled complex gyrator circuit. (b) Frequency response of  $n = 3$  network with nonzero minimas in the reflection coefficient.

the susceptance slope parameter to have a small but non-zero value. This result suggests that the network problem can accommodate some uncertainty in the definition of the coupling angle (susceptance slope parameter and/or loaded  $Q$ -factor) of the junction provided the ripple level of the overall device is bounded by the type of frequency response depicted in Fig. 1(b).

### III. SEMI-TRACKING CIRCULATION SOLUTIONS

Circulator solutions using semi-tracking gyrator circuits may be formed by varying the coupling angle  $\psi$  and/or magnetization of the resonator about the values given by the Wu and Rosenbaum tracking solution. A complete description of this class of device requires a knowledge of the complex gyrator circuit and loaded  $Q$ -factor of the junction. These quantities are evaluated in this section from the description of the complex gyrator admittance of the circulator in the neighbourhood of the first circulation condition of Davies and Cohen [10]. It is also separately necessary to investigate to what extent these variables hold over the desired frequency interval of the device [7]. However, except for the tracking or three eigenvalue boundary conditions [5], a general solution is not currently available in terms of circuit variables; it is therefore necessary to determine them numerically by expanding the two circulation conditions in the vicinity of the

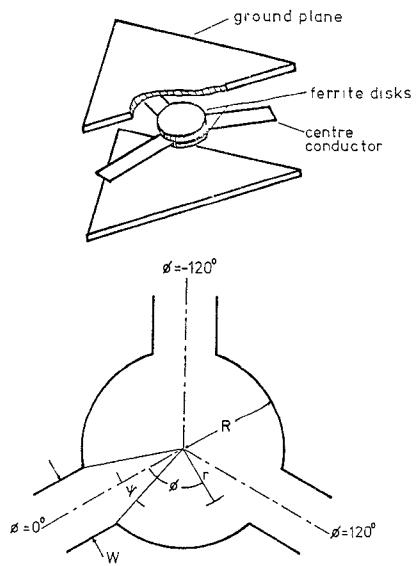


Fig. 2. Schematic diagram of a stripline circulator.

first one. It thus requires a knowledge of the radial number  $kR$  from a statement of the coupling angle  $\psi$  and the magnetic variables  $\kappa$  and  $\mu$  at magnetic saturation. This expansion is also necessary in order to analyse the frequency response of quarter-wave coupled devices.

TABLE I  
DESIGN VARIABLES OF  $n = 3$  SEMI-TRACKING CIRCULATORS ( $S(\max) \neq 1.0$ ,  $S(\min) = 1.0$ )

$S(\max)$	$2\delta_0$	$b'$	$g$	$Q_t$	$y_1$	$y_2$
1.01	0.660	0.252	1.540	0.164	1.073	1.332
1.02	0.660	0.587	2.111	0.278	1.137	1.652
1.03	0.660	0.992	2.702	0.367	1.194	1.962
1.04	0.660	1.456	3.306	0.440	1.245	2.266
1.05	0.660	1.973	3.921	0.503	1.294	2.563
1.06	0.660	2.536	4.541	0.558	1.340	2.855
1.07	0.660	3.140	5.165	0.608	1.382	3.142
1.08	0.660	3.782	5.971	0.653	1.423	3.424
1.09	0.660	4.459	6.418	0.695	1.462	3.703
1.10	0.660	5.166	7.043	0.734	1.499	3.977
1.11	0.660	5.903	7.667	0.770	1.534	4.249
1.12	0.660	6.665	8.288	0.804	1.569	4.517
1.13	0.660	7.452	8.906	0.837	1.602	4.781
1.14	0.660	8.261	9.521	0.868	1.634	5.043

TABLE II  
REALIZABLE SPECIFICATIONS OF  $n = 3$  NETWORKS WITH  $2\delta_0 = 0.66$  ( $S(\max) \neq 1.0$ ,  $S(\min) \neq 1.0$ )

$S(\max) = 1.05$ ,	$S(\min) = 1.005$	$G = 3.58 \text{ Y}_0$ , $B' = 1.740 \text{ Y}_0$
$S(\max) = 1.10$ ,	$S(\min) = 1.07$	$G = 3.62 \text{ Y}_0$ , $B' = 2.423 \text{ Y}_0$
$S(\max) = 1.15$ ,	$S(\min) = 1.12$	$G = 3.76 \text{ Y}_0$ , $B' = 3.010 \text{ Y}_0$
$S(\max) = 1.20$ ,	$S(\min) = 1.17$	$G = 3.82 \text{ Y}_0$ , $B' = 3.244 \text{ Y}_0$

The complex gyrator circuit may be described by

$$G_{in} = \frac{R_{in}}{R_{in}^2 + X_{in}^2} \quad (1)$$

$$B_{in} = \frac{-jX_{in}}{R_{in}^2 + X_{in}^2}. \quad (2)$$

The circulation conditions are defined in the usual way by

$$B_{in} = 0 \quad (3)$$

$$G_0 = G_{in}|_{B_{in}} = 0. \quad (4)$$

$X_{in}$ ,  $R_{in}$ ,  $B_{in}$  and  $G_{in}$  are described in [5].

The first of these equations defines the planar circuit in terms of  $psi$ ,  $\kappa/\mu$  and  $kR$ , and the second describes the absolute conductance level in terms of the admittance ( $Y_r$ ) defined by the resonator terminals. The susceptance slope parameter is determined from a knowledge of  $B_{in}$  in the vicinity of the

first circulation condition

$$B' = \frac{k_0 R}{2} \left[ \frac{B_{in}(\delta) - B_{in}(-\delta)}{k_0 R(1 + \delta) - k_0 R(1 - \delta)} \right]$$

$$\text{or} \quad B' = \frac{B_{in}(\delta) - B_{in}(-\delta)}{4\delta} \quad (5)$$

where

$$\delta = \frac{\omega - \omega_0}{\omega_0}. \quad (6)$$

Finally, the quality factor ( $Q_L$ ) of circuit is calculated by forming

$$Q_L = \frac{B'}{G_0}. \quad (7)$$

Table III depicts semi-tracking solutions with  $\kappa/\mu$  and  $psi$  in the neighborhood of the tracking one. A comparison between the data in this table and that in Table I suggests that such semi-tracking solutions are particularly attractive for the design of octave band devices. In the case of the tracking solution [5], the agreement between the analytical and computed values of

TABLE III  
CIRCULATION CONDITIONS FOR SEMI-TRACKING CIRCULATORS.

psi	kR	G	B'	Q <sub>L</sub>
0.543*	1.6610	0.8880Y <sub>f</sub>	0.4333Y <sub>f</sub>	0.4880
0.550*	1.6453	0.8727Y <sub>f</sub>	0.4480Y <sub>f</sub>	0.5138
0.575*	1.6018	0.8238Y <sub>f</sub>	0.4855Y <sub>f</sub>	0.5894
0.600*	1.5685	0.7841Y <sub>f</sub>	0.5074Y <sub>f</sub>	0.6472
0.625	1.5413	0.7506Y <sub>f</sub>	0.5197Y <sub>f</sub>	0.6924
0.650	1.5183	0.7219Y <sub>f</sub>	0.5253Y <sub>f</sub>	0.7275
0.675	1.4985	0.6971Y <sub>f</sub>	0.5267Y <sub>f</sub>	0.7543
0.700	1.4810	0.6763Y <sub>f</sub>	0.5233Y <sub>f</sub>	0.7739

$\kappa = 0.525, \mu = 1$

psi	kR	G	B'	Q <sub>L</sub>
0.538*	1.6062	0.9302Y <sub>f</sub>	0.3486Y <sub>f</sub>	0.3486
0.550	1.5727	0.9011Y <sub>f</sub>	0.3628Y <sub>f</sub>	0.4021
0.575	1.5241	0.8507Y <sub>f</sub>	0.4118Y <sub>f</sub>	0.4842
0.600	1.4879	0.8098Y <sub>f</sub>	0.5446Y <sub>f</sub>	0.5446
0.625	1.4589	0.7757Y <sub>f</sub>	0.5905Y <sub>f</sub>	0.5905
0.650	1.4347	0.7466Y <sub>f</sub>	0.6254Y <sub>f</sub>	0.6254
0.675	1.4139	0.7216Y <sub>f</sub>	0.4670Y <sub>f</sub>	0.6513
0.700	1.3956	0.7002Y <sub>f</sub>	0.4689Y <sub>f</sub>	0.6697

$\kappa = 0.575, \mu = 1$

psi	kR	G	B'	Q <sub>L</sub>
0.535*	1.5740	0.947Y <sub>f</sub>	0.2772Y <sub>f</sub>	0.2927
0.550*	1.5290	0.9093Y <sub>f</sub>	0.3305Y <sub>f</sub>	0.3634
0.575*	1.4794	0.8598Y <sub>f</sub>	0.3834Y <sub>f</sub>	0.4453
0.600	1.4428	0.8192Y <sub>f</sub>	0.4138Y <sub>f</sub>	0.5051
0.625	1.4136	0.7852Y <sub>f</sub>	0.4319Y <sub>f</sub>	0.5500
0.650	1.3892	0.7561Y <sub>f</sub>	0.4416Y <sub>f</sub>	0.5839
0.675	1.3682	0.7313Y <sub>f</sub>	0.4452Y <sub>f</sub>	0.6089
0.700	1.3498	0.7098Y <sub>f</sub>	0.4445Y <sub>f</sub>	0.6263

$\kappa = 0.600, \mu = 1$

$Q_L$  is in keeping with the interval over which  $B'$  in (5) is evaluated.

Intermediate values of  $\psi$  and  $Q_L$  may be approximated in terms of the first two terms of the Taylor expansion or by some more elaborate interpolation procedure.

$$\psi_i = \psi_{i-1} + \frac{\psi_{i-1} - \psi_{i+1}}{Q_{i-1} - Q_{i+1}} (Q_i - Q_{i-1}) \quad (8)$$

$i$  indicates the required quantities,  $i - 1$  and  $i + 1$  refer to the known quantities.

However, a knowledge of the quality factor although mandatory is not sufficient in that it is also necessary to ensure that the complex gyrator circuit is well behaved over the frequency interval of interest. Fig. 3 illustrates the frequency response of one typical semi-tracking solution. It indicates that

the frequency characteristics of semi-tracking solutions using disk resonators are exceptionally well behaved and are indeed appropriate with the design of octave-band devices.

Its complex gyrator is characterized by

$$G = 0.059 \text{ mhos}$$

$$B' = 0.030$$

$$Q_L = 0.515$$

$$2\delta_0 = 0.65$$

$$Z_r = 50 \Omega$$

The results in Table III marked by an asterisk are not suitable for the design of such devices in that the junctions defined by these boundary conditions exhibit a reversal in the

TABLE III *Cont.*

psi	kR	G	B'	Q <sub>t</sub>
0.531*	1.5384	0.9537Y <sub>f</sub>	0.2433Y <sub>f</sub>	0.2552
0.550	1.4806	0.9151Y <sub>f</sub>	0.3053Y <sub>f</sub>	0.3336
0.575	1.4313	0.8666Y <sub>f</sub>	0.3581Y <sub>f</sub>	0.4133
0.600	1.3949	0.8267Y <sub>f</sub>	0.3897Y <sub>f</sub>	0.4714
0.625	1.3658	0.7930Y <sub>f</sub>	0.4083Y <sub>f</sub>	0.5150
0.650	1.3414	0.7642Y <sub>f</sub>	0.4185Y <sub>f</sub>	0.5475
0.675	1.3205	0.7396Y <sub>f</sub>	0.4224Y <sub>f</sub>	0.5712
0.700	1.3021	0.7183Y <sub>f</sub>	0.4220Y <sub>f</sub>	0.5876

$\kappa = 0.625, \mu = 1$

psi	kR	G	B'	Q <sub>t</sub>
0.550	1.4281	0.9818Y <sub>f</sub>	0.2853Y <sub>f</sub>	0.3107
0.575	1.3799	0.8715Y <sub>f</sub>	0.3372Y <sub>f</sub>	0.3870
0.600	1.3442	0.8324Y <sub>f</sub>	0.3685Y <sub>f</sub>	0.4427
0.625	1.3155	0.7993Y <sub>f</sub>	0.3872Y <sub>f</sub>	0.4844
0.650	1.2915	0.7708Y <sub>f</sub>	0.3972Y <sub>f</sub>	0.5153
0.675	1.2708	0.7465Y <sub>f</sub>	0.4013Y <sub>f</sub>	0.5376
0.700	1.2525	0.7255Y <sub>f</sub>	0.4009Y <sub>f</sub>	0.5527

$\kappa = 0.650, \mu = 1$

psi	kR	G	B'	Q <sub>t</sub>
0.522	1.4650	0.9830Y <sub>f</sub>	0.1735Y <sub>f</sub>	0.1765
0.550	1.3834	0.9200Y <sub>f</sub>	0.2726Y <sub>f</sub>	0.2963
0.575	1.3367	0.8744Y <sub>f</sub>	0.3228Y <sub>f</sub>	0.3692
0.600	1.3017	0.8359Y <sub>f</sub>	0.3533Y <sub>f</sub>	0.4227
0.625	1.2737	0.8034Y <sub>f</sub>	0.3717Y <sub>f</sub>	0.4626
0.650	1.2501	0.7755Y <sub>f</sub>	0.3816Y <sub>f</sub>	0.4921
0.675	1.2296	0.7513Y <sub>f</sub>	0.3855Y <sub>f</sub>	0.5131
0.700	1.2115	0.7303Y <sub>f</sub>	0.3850Y <sub>f</sub>	0.5272

$\kappa = 0.670, \mu = 1$

direction of the circulation at the high frequency end of the band.

Although the synthesis of extended octave-band devices is outside the scope of this paper it may be worthwhile to observe that well behaved semi-tracking gyrator circuits may be realized with such boundary conditions. Fig. 4 illustrates one such solution. Its complex gyrator is characterized by

$$G = 0.0456 \text{ mhos}$$

$$B' = 0.2605$$

$$Q_L = 0.626$$

$$2\delta_0 = 0.70$$

$$Z_r = 60 \Omega.$$

The physical variables in the tables may be understood by constructing an example. One possibility is defined by

$$\kappa = \frac{\omega_m}{\omega} \approx 0.67 \quad (9)$$

$$\mu = 1 \quad (10)$$

$$\psi \approx 0.522 \quad (11)$$

$$kR \approx 1.465 \quad (12)$$

The absolute gyrator conductance defined by the above variables is given by

$$G = 0.9822Y_f \quad (13)$$

where

$$Y_f = Y_r \sqrt{\varepsilon_f} \quad (14)$$

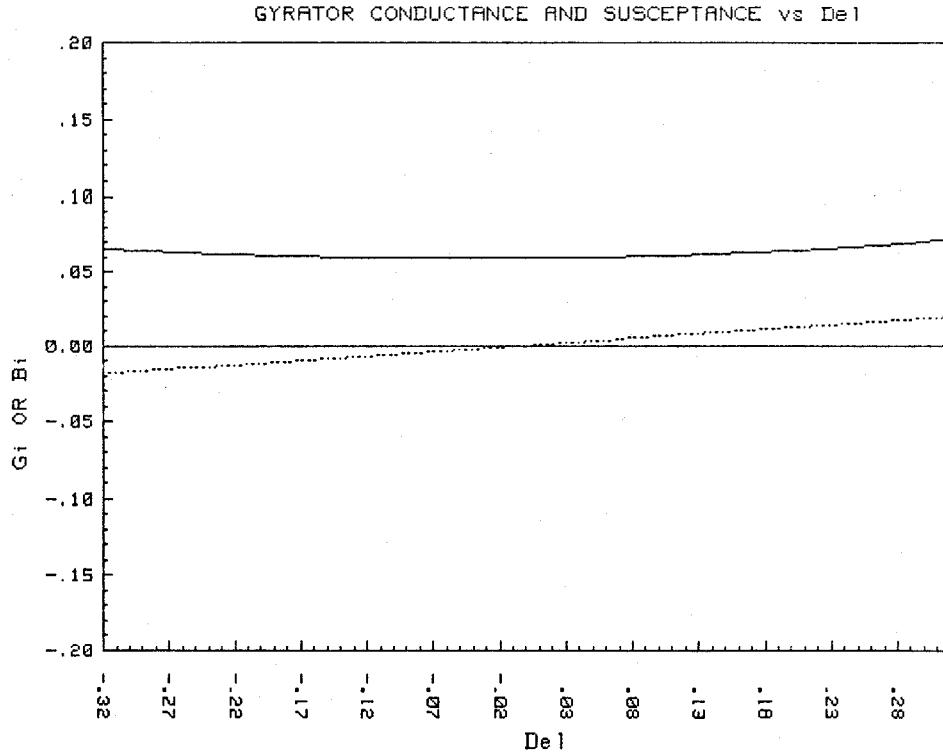


Fig. 3. The real and imaginary parts of the complex gyrator circuit with  $\mu = 1$ ,  $\kappa = 0.65$ ,  $\psi = 0.65$  rad,  $Z_r = 50 \Omega$ ,  $kR = 1.291$ .

and

$$Y_r \approx \left[ 30\pi \ln \left( \frac{W+t+2H}{W+t} \right) \right]^{-1}. \quad (15)$$

$Y_r$  in (14) and (15) defines the free space conductance of the three transmission lines adjacent to the junction. In a directly coupled junction it is usually chosen as 0.02 mhos, but in a quarter-wave coupled one, with  $(n-l)$  transformer sections, it may be used to adjust the gyrator conductance by varying  $H$  in order to satisfy the network problem, and the dielectric constant  $\epsilon_t$  of the dielectric region adjacent to the junction may be employed to set the admittance  $Y_{n-1}$  of the quarter-wave transformer adjacent to the junction.

$$Y_{n-1} = Y_r \sqrt{\epsilon_t}. \quad (16)$$

$kR$  and  $\psi$  are defined in the standard way

$$kR = \frac{2\pi\sqrt{\mu_e\epsilon_f}}{\lambda_0} R \quad (17)$$

$$\sin \psi = \frac{W}{2R}. \quad (18)$$

$\mu_e$  and  $\omega_m$  have the usual meanings and  $W$ ,  $2R$  and  $\psi$  are defined in Fig. 2;  $H$  is the thickness of each disk,  $t$  is the center conductor thickness,  $\epsilon_f$  is the dielectric constant of the ferrite material.

#### IV. DIELECTRIC CONSTANT OF TRANSFORMER REGION

An important quantity in the design of junction circulators is the value of the dielectric constant of the region adjacent to the resonator. Unfortunately, this quantity is not an independent

variable, but is dependent upon both the overall specification of the device and the semi-tracking solution adopted for its realization. The evaluation of this quantity will now be demonstrated by way of an example.

The absolute entries of the network problem for  $VSWR(\max) = 1.04$  and  $2\delta_0 = 0.66$  are obtained from Table I as

$$\begin{aligned} G &= 0.0661 \\ B' &= 0.0291 \\ Q_L &= 0.4407 \\ Y_1 &= 0.0249 \\ Y_2 &= 0.0453. \end{aligned}$$

Table III indicates that the boundary conditions required to satisfy this value of  $Q_L$  are not unique. One solution is

$$\begin{aligned} \psi &= 0.600 \\ kR &= 1.3442 \\ G &= 0.8324Y_f \\ B' &= 0.3685Y_f. \end{aligned}$$

$Y_f$  may be evaluated using either  $G$  or  $B'$ . In the procedure adopted here it is obtained by reconciling the two statements for  $G$ .

Taking  $\epsilon_f$  as 14.7 and making use of the definition of  $Y_f$  in (14) leads to

$$G = 3.206Y_r$$

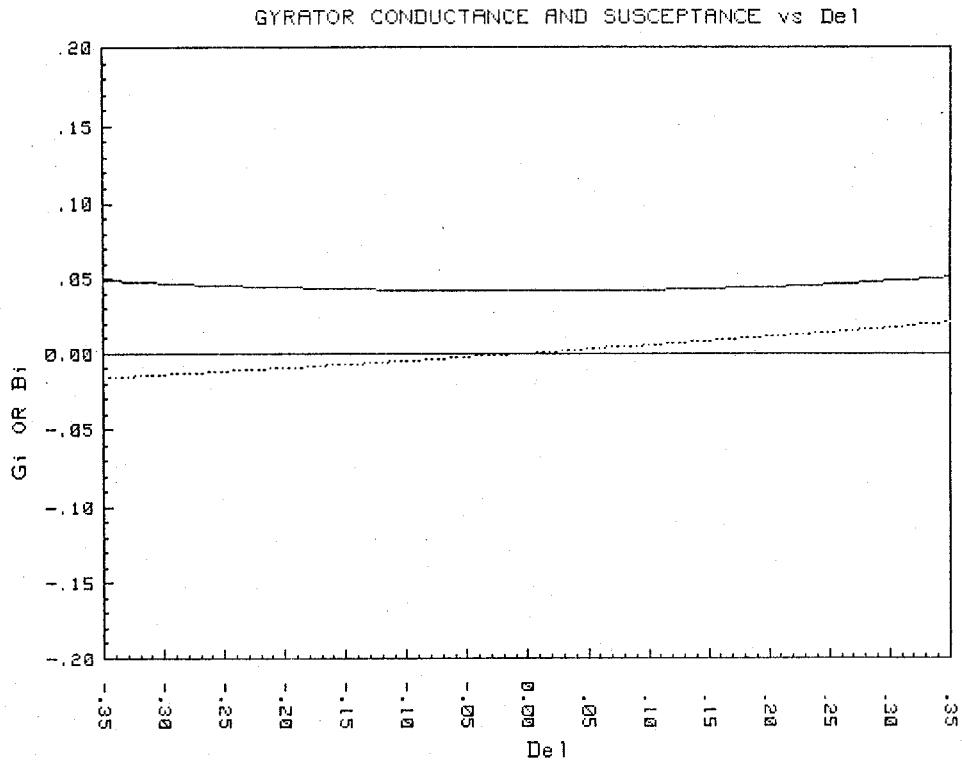


Fig. 4. The real and imaginary parts of the complex gyrator circuit with  $\mu = 1$ ,  $\kappa = 0.60$ ,  $\psi = 0.70$  rad,  $Z_r = 60 \Omega$ ,  $kR = 1.350$ .

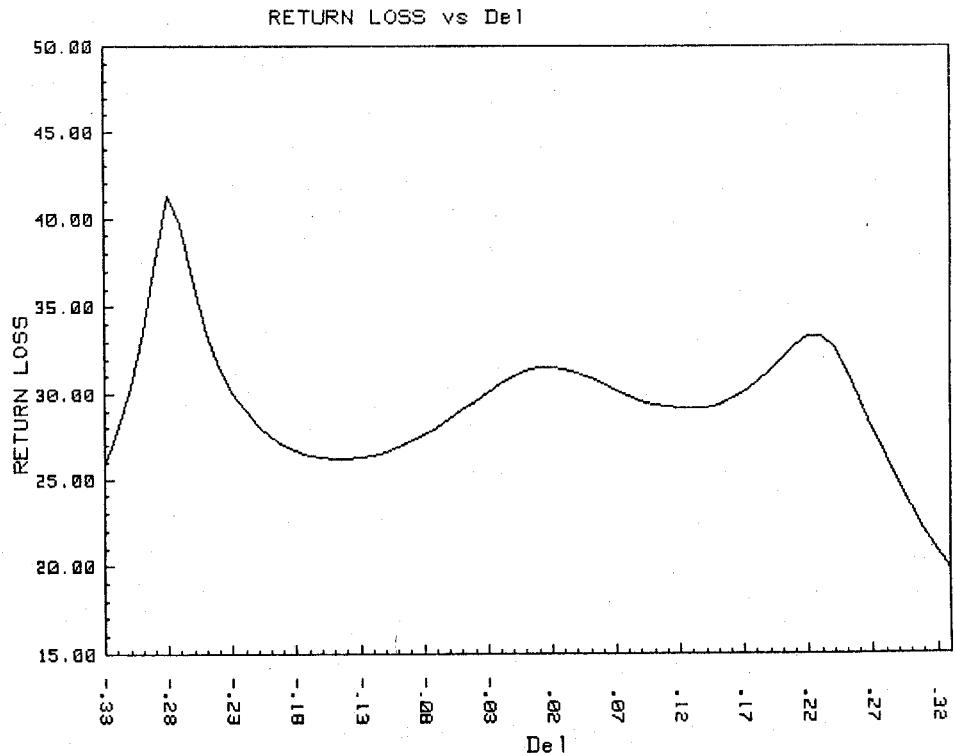


Fig. 5. Theoretical frequency response of two section quarter-wave coupled semi-tracking junction with  $\mu = 1$ ,  $\kappa = 0.60$ ,  $\psi = 0.65$ ,  $G = 0.0780$ ,  $B' = 0.0467$ ,  $Q_L = 0.584$ ,  $Z_r = 36.25$ ,  $\Omega$ ,  $\epsilon_d = 3.96$ .

Equating the values of  $G$  determined by the network and e.m. problems gives the following relationship

$$0.06612 = 3.406Y_r.$$

Solving for  $Y_r$  yields

$$Y_r = 0.0206.$$

$\epsilon_d$  is now determined from a knowledge of  $Y_r$  and  $Y_{n-1}$  with the aid of (16). The result is

$$\epsilon_d = \left( \frac{Y_2}{Y_r} \right)^2 = 4.8.$$

The nearest integer value for  $\epsilon_d$  is 5. If  $\epsilon_d$  does not correspond to an integer or commercial value the design must be repeated

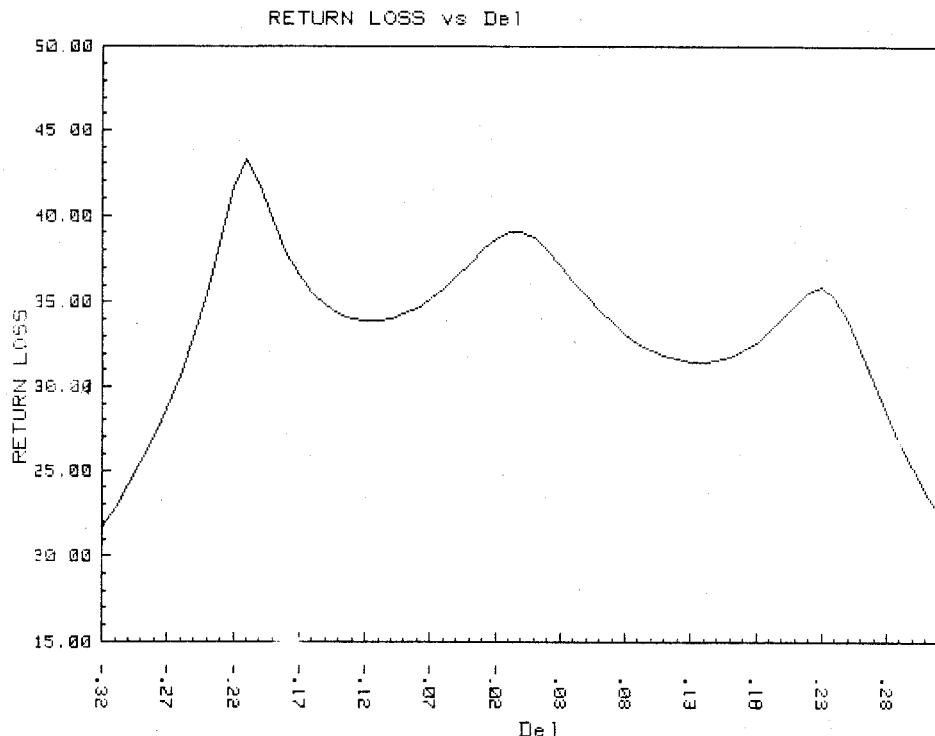


Fig. 6. Theoretical frequency response of two section quarter-wave coupled semi-tracking junction with  $\mu = 1$ ,  $\kappa = 0.67$ ,  $\psi = 0.70$ ,  $G = 0.077$ ,  $B' = 0.0404$ ,  $Q_L = 0.527$ ,  $Z_r = 36.50 \Omega$ ,  $\epsilon_d = 3.50$ .

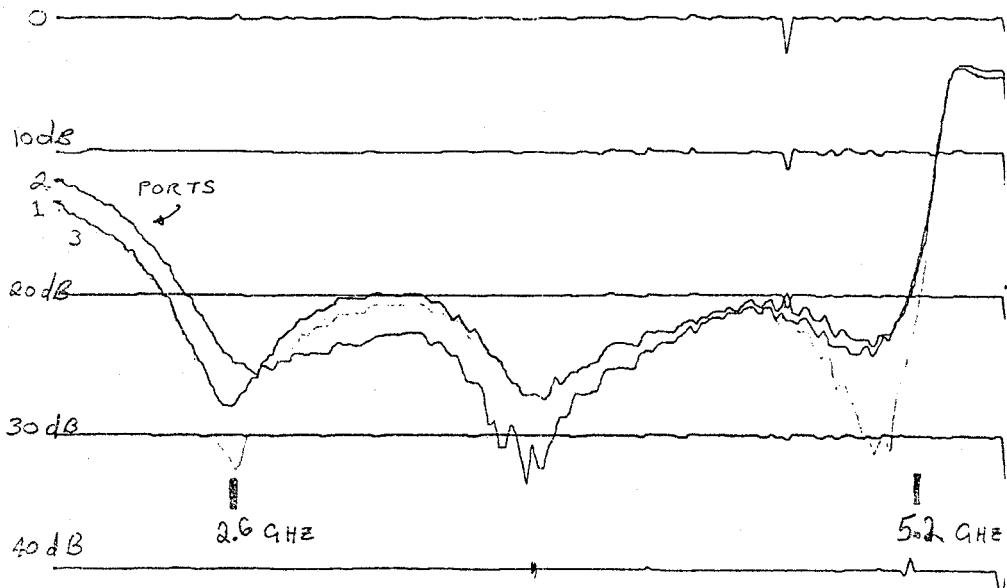


Fig. 7. Experimental performance of the two-section, quarter-wave coupled, octave-band, circulator.

with a different ripple level or with a different choice of semi-tracking solution in Table III.

The width of the striplines adjacent to the resonator is calculated next in terms of  $\psi$  once the radius of the resonator has been formed from a knowledge of the center frequency of the device and  $kR$ . Finally, the thickness  $H$  of the resonators is evaluated from a knowledge of  $W$  and  $Y_r$ .

One feature of the ideal semi-tracking solution of circulators using disk resonators is that the dielectric constant of the transformer region adjacent to the resonator lies ap-

proximately between 3 and 6. The use of such low values of dielectric constant adjacent to the resonator readily reproduces the assumed magnetic wall boundary condition between the ports of the junction. It therefore more readily ensures correlation between practice and theory than would be the case if it were to be comparable with that of the ferrite or garnet material. Failure to accurately reproduce the boundary conditions between the ports of the junction leads to some uncertainty in the definition of the effective coupling angle of the junction (defined by the transmission

lines and the resonator circuit) and of the radius of the resonator. There is also some corresponding modification in the susceptance slope parameter and to a lesser extent in the conductance of the complex gyrator circuit. Fortunately, the field of solutions of the semi-tracking subspace outlined here permits some laxity in the definition of the former parameters and the network problem can accommodate some uncertainty in the latter quantities if the minimas in the reflection coefficient are not forced to pass thru zero [9].

## V. DESIGN OF OCTAVE-BAND SEMI-TRACKING CIRCULATORS

A perusal of Tables I and III indicates that the quality factors of semi-tracking junctions are compatible with the realization of octave-band circulators with VSWR's between about 1.03 and 1.10. It is also observed from this data that the quality factor of the gyrator circuit is not unique in that some trade-off is possible between the magnetization and the coupling angle. In general, the use of wide coupling angles and large values of magnetization moves the frequency at which the direction of circulation reverses in this type of junction outside the required pass-band of the specification, and additionally, loosely speaking, the more optimum frequency responses. The synthesis of devices with very low ripple levels is of course quite demanding upon the quality of the equivalent circuit, but by and large it appears to be good enough for the realization of devices for which the VSWR is below 1.20 at the bandedges. Figs. 5 and 6 illustrate two typical responses.

Fig. 7 illustrates the performance of a 2.6–5.2 GHz device developed prior to the development of the full theory. The e.m. problem was determined by taking the tracking solution as a trial function and the matching network was formed by assuming a return loss of 20 dB for the overall network. The ripple characteristic is in keeping with the fact that the coupling angles in Table III represent a lower bound on the effective ones. Although the agreement between theory and experiment is not complete it is certainly within measurement, connector and material tolerance. The only significant discrepancy is the use of a dielectric material with a dielectric constant of 7 instead of 4.80 for the transformer region. However, analysis suggests the possibility of realizing non-optimum responses at the 20 dB points using a dielectric constant of say 7. In obtaining this data the applied direct field was set experimentally and some rework of the first transformer was also carried out.

## VI. CONCLUSION

The synthesis of quarter-wave coupled stripline junction circulators with equal ripple reflection coefficients biased below the main kittel resonance line requires junctions with semi-tracking boundary conditions for their realizations. This paper describes a complete numerical formulation of this problem as a preamble to the study of this class of devices. Useful results for design are presented in tabular form. The overall frequency response of this type of circulator has been evaluated and found in agreement with the idealized model employed here.

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**J. Helszajn** (M'64–SM'87–F'92), for a biography and photograph, see p. 1958 of the November 1993 issue of this TRANSACTIONS